IV-1. SYNTHESIS OF DISTRIBUTED ELLIPTIC-FUNCTION FILTERS FROM LUMPED-CONSTANT PROTOTYPES

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Filters based on the elliptic-function or Cauer-parameter type of response give rapid cut-off compared with filter responses of Chebychev or Butterworth types which have no attenuation poles at finite frequencies. The response of a distributed band-stop filter consisting of commensurate lengths of transmission line, based on a two-pole lumped element prototype, is shown in Fig. 1 as an example of the type of response desired.

Although elliptic-function filters have been developed and are used extensively at low frequencies using lumped components, there has been little application of the technique in the microwave band, using distributed circuit elements. An early reference to a design procedure is contained in the paper of Ozaki and Ishii[1], their method consisting of a direct synthesis from the network function, but little design data are given. An interesting procedure based on coupled transmission lines was introduced by Saito[2], but is subject to limitations due to extreme impedance levels. A multi-harmonic rejection filter designed from an elliptic-function prototype has been described by Schiffman[3], but is limited to prototypes having not more than one transmission zero at a finite frequency.

Although direct synthesis of distributed elliptic-function filters is certainly possible[1], a great deal of work is involved in the preparation of suitable computer programs. As an alternative it was decided to take advantage of the considerable effort which has led to the publication of comprehensive tables for lumped-element elliptic-function filters[4]. The distributed filters described in this paper use the low-pass prototypes of these tables. The prototype is converted by means of Richards' transformation into a distributed-constant filter consisting of a number of commensurate stubs located at one physical point. These stubs must now be separated by cascaded lengths of transmission line (unit elements) to give a filter which is reasonable in practice. A method of stub separation which is used frequently in the case of certain restricted classes of ladder networks is by the application of the well-known Kuroda identities. These establish an exact equivalence between a unit element in cascade with a series or shunt open- or short-circuited stub on the one hand, and the reversed cascade consisting of a stub followed by a unit element on the other. Since it is possible to incorporate any number of unit elements at either port of the ladder network, each of characteristic impedance equal to the terminating resistance at the respective ports without altering the insertion loss, then it is possible to transform these unit elements into the network by successive applications of the Kuroda identities in such a way as to separate the stubs. The restriction on this method is that the simple Kuroda identities apply only to single stubs, equivalent to a series or shunt capacitor or inductor of the lumped-element prototype. However it has been shown[5] that the simple Kuroda identities are special cases of

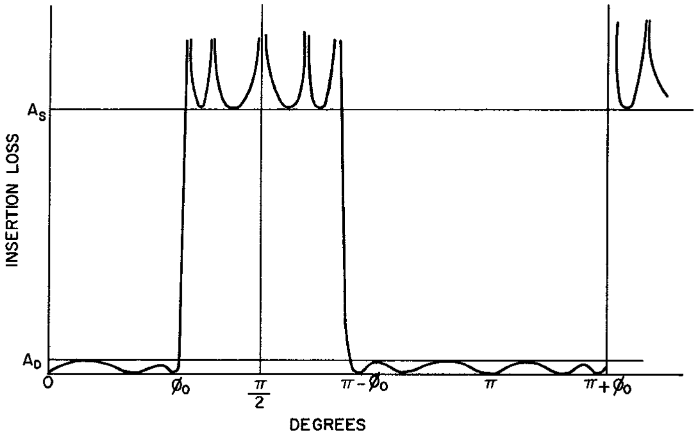


Figure 1. Response of a Distributed Elliptic-Function Band-Stop Filter

a more general transformation which may be stated as a theorem:

"A distributed network consisting of a unit element of characteristic impedance Z in cascade with a physically realizable two-port N may always be replaced by an equivalent circuit consisting of a physically realizable two-port N' in cascade with a simple unit element of characteristic impedance Z'."

Thus if the first network N following the unit element is defined by its transfer (or chain) matrix

$$\begin{bmatrix} A(t) & B(t) \\ C(t) & D(t) \end{bmatrix} \tag{1}$$

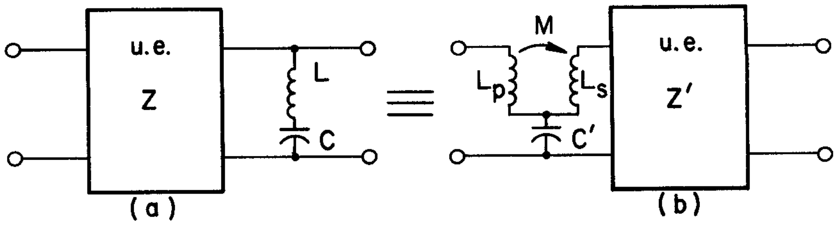
where the matrix elements are functions of the complex distributed frequency variable t, then in the equivalent circuit the network N' has the transfer matrix

$$\frac{1}{1-t^2}\begin{bmatrix} A + CZt - \frac{B}{Z'}t - \frac{DZ}{Z'}t^2 & -AZ't - CZZ't^2 + B + DZt \\ C + \frac{A}{Z}t - \frac{D}{Z'}t - \frac{B}{ZZ'}t^2 & -CZ't - \frac{AZ'}{Z}t^2 + D + \frac{B}{Z}t \end{bmatrix} \tag{2}$$

and the unit element Z' is given by

$$Z' = \frac{B(1) + ZD(1)}{A(1) + ZC(1)} \tag{3}$$

In the application of this generalized transformation to elliptic-function filters, it is frequently required to transform a unit element across a distributed series resonant circuit in shunt (that is, a shunt Foster) as shown in Fig. 2a. The result of the transformation is a microwave Brune section in cascade with a unit element as shown in Fig. 2b. If a further transformation of a unit element is required, this will be across the



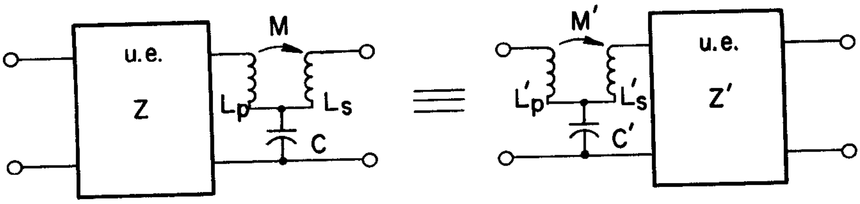
$$Z' = \frac{Z(1 + LC)}{1 + C(Z + L)} \qquad C' = \frac{LC^2}{1 + LC}$$

$$L_p = \frac{1 + C(Z + L)}{C} \qquad M = \frac{1 + LC}{C}$$

$$M^2 = L_p L_s$$

Figure 2. Transformation of Unit Element across Distributed Series-Resonant Circuit in Shunt

microwave Brune section, which is again accomplished by means of the generalized transformation. It is easy to show that the transformation of a unit element across a Brune section results in another Brune section, as shown in Fig. 3. It is evident that this depicts a more general case than that shown in Fig. 2, where the shunt Foster of Fig. 2a is to be regarded as a degenerate Brune section. The equation in Fig. 2 may be derived from those of Fig. 3 by substituting the conditions for degeneracy, i.e., that $L_p = L_s = M$.



$$Z' = \frac{Z(L_p + M^2 C) + (L_p - M)^2}{L_p\left[1 + C(Z + L_p)\right]}$$

$$C' = C + \frac{1}{Z} - \frac{1}{Z'} \qquad M'C' = MC$$

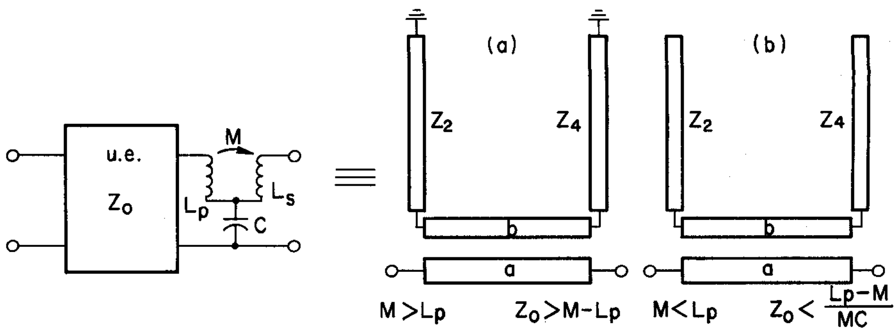
$$L_p' = \frac{ZCM^2}{Z'C'L_p} \qquad L_s' = \frac{Z'CL_p}{ZC'}$$

Figure 3. Transformation of Unit Element Across Brune Section

Realizations of Microwave Brune Sections. A considerable amount of work on the physical realization of microwave C, Brune, and D sections has been accomplished by several workers. Only Brune sections are required in the realization of elliptic-function filters, since the latter has only real frequency transmission zeros. At present no realization exists for the microwave Brune section alone, but there are several realizations for the microwave Brune section in cascade with a unit element, e.g., Ikeno loop[6] or the Saito section[7], which are subject to severe restrictive conditions on their realizability. These were released to some extent by Matsumoto[7] who gave six types of realizations using three-wire lines. The formulae given by Matsumoto contain minor errors, which have been corrected by Rhodes and Scanlan[8] who have shown that it is not possible to realize a Brune section in cascade with a unit element of arbitrary impedance by a choice of one of Matsumoto's configurations. In practice it is found that some of his networks are difficult to construct, while the three-wire configuration is difficult to calculate as far as the self and mutual capacitance values of the wire are concerned. It has been found more convenient, therefore, to use new physical realizations using two-wire coupled lines and stubs, as shown in Fig. 4. The circuit with short-circuited stubs in Fig. 4 applies for $M > L_p$, and has restrictive conditions identical to those of the Ikeno loop or the Saito section. With open-circuited stubs, cases for which $M < L_p$ may be realized, subject to a restrictive condition on Z_o, i.e., that it may not be larger than $(L_p - M)/MC$. It may appear from the restrictive conditions on Z_o that these coupled-line circuits are of limited use. However, in practice the restrictions are not serious, since Scanlan and Rhodes[8] have shown that given a prescribed physically-realizable driving point impedance with transmission zeros all at real frequencies, it is always possible to find a physical realization consisting of a cascade of unit elements and Brune sections having these restrictions (which appear to be basic). Thus, in a synthesis procedure it may not be possible to extract a Brune section at some stages, but by first extracting one or more unit elements the eventual extraction of the Brune section for the particular transmission zero is guaranteed. In practice it is rarely necessary to extract more than one or two unit elements before the Brune section may be realized.

Several elliptic-function microwave filters have been designed and manufactured using microwave Brune sections. In the design procedure the most important requirement is that the impedances of all lines within the filters have values which can readily be made in practice, i.e., lying in the range 10 to 200 ohms. In this context it should be noted that physical realizability criteria in the strict circuit theory sense are not sufficient for actual practical realizations because they do not exclude structures with impossible impedance levels.

A typical filter is shown in Fig. 5. It is a stop-band filter with a stop-bandwidth to the 40.2 dB points of 2.85%, and a comparison of the theoretical and experimental performance is shown in Fig. 6. The filter consists of five unit element/Brune sections in cascade. Actually in this case the Brune sections all degenerate to simple series or shunt Foster sections, which in practice means that in each case one of the two stubs of the Brune sections shown in Fig. 4 degenerates to an open or short circuit. Filters employing the general Brune section have been constructed, and the results will be presented.



$$Z_b = \frac{Z_{oe}^b + Z_{oo}^b}{2}, \cdot \text{ is a parameter} \qquad \frac{Z_{oe}^a + Z_{oo}^a}{2} = \frac{M Z_o}{L_p}$$

(a)

$$\frac{Z_{oe}^a - Z_{oo}^a}{2} = \frac{Z_{oe}^b - Z_{oo}^b}{2} = \sqrt{\frac{(MCZ_o + M - L_p)Z_o Z_b}{L_p(CZ_o + 1)}}$$

$$Z_2 = \frac{C L_p Z_o Z_b}{(M - L_p)(CZ_o + 1)} \qquad Z_4 = \frac{(MCZ_o + M - L_p) Z_b}{(Z_o + L_p - M)(C Z_o + 1)}$$

(b)

$$\frac{Z_{oe}^a + Z_{oo}^a}{2} = \frac{Z_o + (L_p - M)^2}{L_p} \qquad Z_4 = \frac{(L_p - M - MCZ_o) Z_b}{L_p C(L_p - M + Z_o)}$$

$$\frac{Z_{oe}^a - Z_{oo}^a}{2} = \frac{Z_{oe}^b - Z_{oo}^b}{2} = \sqrt{\frac{(L_p - M)(Z_o + L_p - M) Z_b}{L_p}} \qquad Z_2 = \frac{Z_o Z_b}{(L_p - M)}$$

Figure 4. Realizations of Microwave Brune Sections Using Two-Wire Coupled Lines and Stubs

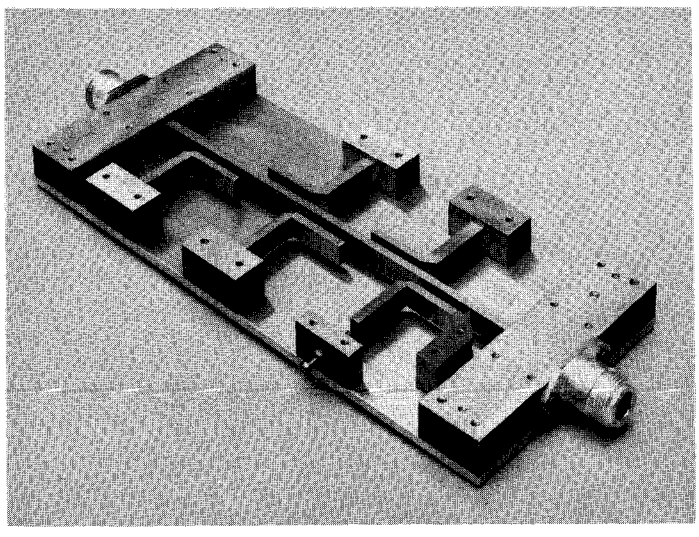


Figure 5. Typical Elliptic-Function Stop-Band Filter

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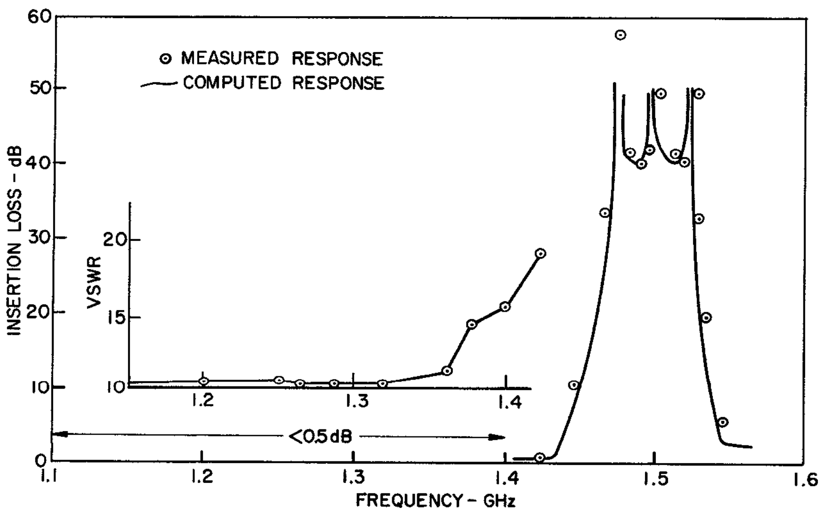

Figure 6. Theoretical and Experimental Responses of Stop-Band Filter